Effect of Lift with Roll Rate Variation on Re-Entry Vehicle Impact

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The effect of lift with roll rate variation on the impact of re-entry vehicles is treated analytically. The equations of motion of a vehicle experiencing lunar motion at small angle of attack are integrated assuming the roll rate history can be approximated with a series of linear segments. Each segment occurs over a sufficiently short interval of time that the trajectory properties can be assumed constant. The result is a closed-form solution expressing the resulting impact deviation from the zero-lift case as a function of the zero-lift trajectory characteristics and the roll rate history. The effect of a single linear roll rate variation between arbitrary initial and final roll rates on impact is derived from this general result. Roll through zero between roll rates of large magnitude but opposite sign, and roll acceleration of infinite magnitude between arbitrary roll rates are presented as special cases. An example problem involving a single linear variation is treated and compared with results obtained using a six-degree-of-freedom trajectory simulation program. These results compare to within 5% typically, with none of the differences exceeding 13%.

Nomenclature†

A = reference area, ft²
$C_{L\alpha}$ = vehicle lift-coefficient-curve slope, /deg
CR = cross range, ft
C_D = vehicle drag coefficient
$C(\ )$ = Fresnel's cosine integral [Eq. (34)]
c.g. = vehicle center of gravity
D = drag force, C_DqA, lb
DR = downrange, ft
exp ( ) = exponential function, $e^{(\ )}$
g = gravitational acceleration, 32.2 ft/sec²
h = altitude, ft
i,j,k = unit vectors in the x, y, and z directions, respectively
I_r = moment of inertia about the vehicle roll axis, slug-ft²
L = lift force, $C_{L\alpha}\alpha qA$, lb
n = total number of linear roll rate variations composing a particular roll rate history
P = roll rate, rad/sec
q = dynamic pressure, $\rho V^2/2$, lb/ft²
r = radius of curvature of motion produced by lift, ft
R = radius of the base of a conical surface of possible dispersed trajectories, ft
$S(\ )$ = Fresnel's sine integral [Eq. (34)]
t = time, sec
V = vehicle velocity, fps
W = vehicle weight, lb
x,y,z = moving coordinates
X,Y,Z = Newtonian coordinates
α = trim angle of attack, deg
β = ballistic coefficient W/C_DA, lb/ft²
γ = flight-path angle (negative quantity), rad
$\bar{\gamma}$ = mean flight-path angle defined by Eq. (16) (negative quantity), rad
δ = angular flight-path deflection, rad
ϵ = angle between the radius vector from the center of curvature of the motion produced by lift and the lift vector, rad
η = +1 for roll acceleration, −1 for roll deceleration [Eq. (38)]
θ = roll angle between the vehicle lift vector and the x axis, $\varphi - \epsilon$, rad
ρ = atmospheric density, slug/ft³
φ = angle between the radius vector from the center of curvature of the motion produced by lift and the x axis, rad

Subscripts

a,b = beginning and end of a linear roll rate variation
av = average condition during roll rate variation
e = re-entry conditions
i,j = counting indices
id,Im = impact deviation and impact, respectively
max = maximum
0 = conditions at instant of roll rate variation
$P = 0$ = condition when roll rate is zero

Superscript

$\dot{(\ )}$ = differentiation with respect to time

Received October 9, 1970; revision received February 17, 1971.
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Introduction

THE aerodynamic response of a re-entry vehicle to shape or mass asymmetries is to assume a trim angle of attack (α). Assuming these asymmetries are fixed relative to the vehicle, the lift force (L) derived from the trim will also be vehicle-fixed. The lift is necessary to balance the torque provided the vehicle by the asymmetry. Since the moment arm from L to the vehicle center of gravity is usually only a small fraction of the vehicle length, the lift necessary to counterbalance this torque is typically large compared to the force that produces the torque. As a result, L is an essentially unbalanced force that causes acceleration away from the zero-lift flight path.

Over the altitude range that roll rate variation is most important to impact (i.e., $h < 100$ Kft), the response frequency of the vehicle to angle of attack is generally much greater than the roll rate (P). When this is the case, unless the torque-producing asymmetry disappears in some manner, the resulting angle of attack α will not converge and disappear in contrast to an initial, or re-entry, angle of attack. Further, after the initial angle of attack has converged and only α remains, the vehicle response in pitch and yaw will prevent the roll rate from rotating the vehicle meridian containing α out of the wind. That is, the vehicle windward meridian will always be the meridian that contains α, just as it would be if

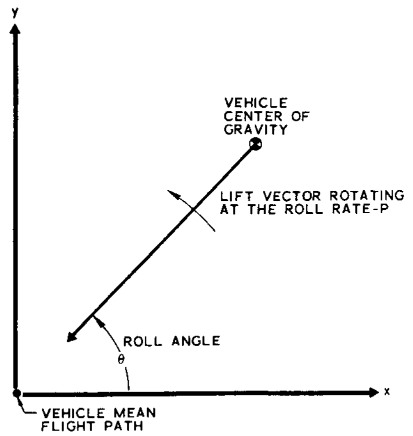

Fig. 1 Re-entry vehicle motion with lift and constant roll rate.

the vehicle were not rolling at all. Because it is rolling, however, the vehicle-fixed lift force is rotated about the mean flight path at the rate of P, and the resulting vehicle acceleration is similarly distributed. Unless the roll rate is zero or very close, constant lift at constant P, therefore, results in a spiral trajectory about a mean flightpath without significant net deviation from the zero-lift trajectory. If P varies, however, the lift is not distributed for an equal amount of time in all directions about the zero-lift flightpath and deviation of the mean flight path from the zero-lift flightpath will occur. The case of constant lift and constant $P = 0$ has been treated previously[1] and is not considered here.

The motion of a re-entry vehicle rolling with a vehicle-fixed windward meridian is commonly termed lunar. The purpose of this study is to consider the effect of roll rate variation and lift on re-entry vehicle impact. An important special case of this general problem is a variation of P that includes $P = 0$. This case has been treated in Ref. 2, assuming the initial and final boundary conditions on P are unimportant to impact. This assumption is valid provided certain conditions on the magnitudes of the initial and final roll rates are met, which is generally the case. The present analysis treats P variations that do not necessarily include $P = 0$ and includes the effects of the boundary conditions on the results.

Motion at Constant Roll Rate

Suppose that a re-entry vehicle experiences lunar motion with a small α (i.e., $\sin\alpha \approx \tan\alpha \approx \alpha$). Assume that the

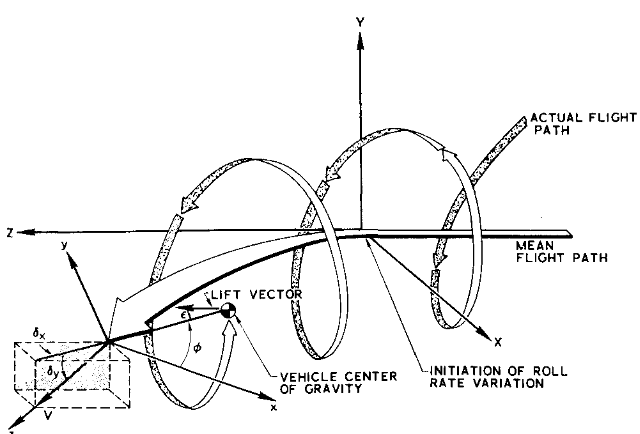

Fig. 2 Re-entry vehicle motion with lift and roll rate variation.

oscillatory portion of the angle of attack has dampened to the trim value, that L and P are both constant, and that P is much less than the response frequency of the vehicle in pitch and yaw but not close to zero. The resulting vehicle motion will include rotation of the c.g. about a mean flightpath, as illustrated in Fig. 1, and translation down this flightpath. The direction of the lift is toward the mean flightpath and is balanced by the centrifugal acceleration of the revolving vehicle. That is,

$$L = C_{L\alpha}\alpha q A = (W/g)P^2 r \quad (1)$$

where r is the distance from the vehicle c.g. to the mean flightpath. The components of position and velocity relative to the nonrotating coordinate system of Fig. 1 are $x = r\cos\theta$, $y = r\sin\theta$; and $\dot{x} = -Pr\sin\theta$, $\dot{y} = Pr\cos\theta$. Using Eq. (1) to obtain the radius of curvature in these expressions yields

$$x = g(L/W)(\cos\theta/P^2), \quad y = g(L/W)(\sin\theta/P^2) \quad (2)$$

and

$$\dot{x} = -g(L/W)(\sin\theta/P), \quad \dot{y} = g(L/W)(\cos\theta/P) \quad (3)$$

The increase in the vehicle drag caused by lift is a negligible fraction of the zero-lift drag provided the magnitude of α satisfies the small-angle relation already assumed. Further, since L is constant (i.e., variations of q and α are small from roll cycle to roll cycle), the resulting acceleration will be averaged about the mean flightpath. Therefore, lift will not cause any significant deviation of the mean flightpath from the zero-lift flightpath for this case.

Motion with Roll Rate Variation

For the purpose of this analysis, the mean flightpath is defined mathematically to be the locus of instantaneous centers of curvature of the motion produced by lift as the vehicle proceeds through the atmosphere. Suppose that although L remains constant, P varies in some prescribed manner. Since the lift is no longer distributed for an equal amount of time in all directions about the center of curvature, there will be lateral motion of the center of curvature as well as motion about the center of curvature as the variation proceeds. This motion relative to a Newtonian coordinate system (X,Y,Z) is illustrated in Fig. 2. The angles δ_x and δ_y represent deflections of the mean flightpath resulting from lift. The x-y plane contains the lift vector and the motion about the center of curvature. The velocity V is the velocity of the instantaneous center of curvature as the vehicle proceeds down its flightpath.

The magnitude of the velocity that the vehicle retains from its value at re-entry is normally far greater than those components caused by lift. A reasonable assumption, therefore, is that lift serves only to change the direction of the velocity vector by small angles without significantly affecting its magnitude. That is,

$$|\delta_x|, |\delta_y| \ll 1; \quad |\dot{x}|/V, |\dot{y}|/V \ll 1 \quad (4)$$

Within these assumptions the velocity of the instantaneous center of curvature is equal in magnitude to the vehicle velocity assuming zero lift. If **i**, **j**, and **k** are unit vectors along the x, y, and z axes, respectively, the equations of motion neglecting the accelerations resulting from gravity and Earth curvature are

$$\mathbf{F} = (W/g)\mathbf{a} \quad (5)$$

where

$$\mathbf{F} = -L\cos(\varphi - \epsilon)\mathbf{i} - L\sin(\varphi - \epsilon)\mathbf{j} - D\mathbf{k} \quad (6)$$

$$\mathbf{a} = [\ddot{x} + (V + y\dot{\delta}_y - x\dot{\delta}_x)\dot{\delta}_x]\mathbf{i} + [\ddot{y} - (V + y\dot{\delta}_y - x\dot{\delta}_x)\dot{\delta}_y]\mathbf{j} + [\dot{V} + y\ddot{\delta}_y - x\ddot{\delta}_x + 2(\dot{y}\dot{\delta}_y - \dot{x}\dot{\delta}_x)]\mathbf{k} \quad (7)$$

Assuming the lift is fixed relative to the vehicle, the angle $\theta = \varphi - \epsilon$ is the vehicle roll angle specified by the prescribed roll rate variation by integrating

$$\ddot{\theta} = d^2(\varphi - \epsilon)/dt^2 = \dot{P} \tag{8}$$

In addition, neglecting the terms of Eq. (7) derived from lift-induced rotation of the x-y-z coordinate system relative to the Newtonian coordinate system compared to V and $\dot{V}$ yields

$$\mathbf{a} = (\ddot{x} + V\dot{\delta}_x)\mathbf{i} + (\ddot{y} - V\dot{\delta}_y)\mathbf{j} + \dot{V}\mathbf{k} \tag{9}$$

Since the origin of the x-y-z coordinate system has been specified as the instantaneous center of curvature of the motion produced by lift, the acceleration vector can be expressed

$$\mathbf{a} = -[d(y\dot{\varphi})/dt]\mathbf{i} + [d(x\dot{\varphi})/dt]\mathbf{j} + \dot{V}\mathbf{k} \tag{10}$$

where

$$\varphi = \tan^{-1}(y/x) \tag{11}$$

within the assumptions of Eq. (9). Equations (5, 6, 9, and 10) can be combined and expressed in scalar form as

$$\begin{aligned} \ddot{x} + V\dot{\delta}_x &= -g(L/W)\cos\theta = -d(y\dot{\varphi})/dt \\ \ddot{y} - V\dot{\delta}_y &= -g(L/W)\sin\theta = d(x\dot{\varphi})/dt \end{aligned} \tag{12}$$

$$\dot{V} = -g(D/W) \tag{13}$$

Within the assumptions of this analysis, the significant impact deviation which results from roll rate variation is derived from the deflection of the mean flightpath that occurs, as opposed to the change in the motion about that flightpath. Equations (12) and (13) will be used to obtain expressions for the impact deviation resulting from such a deflection of the mean flightpath.

For straight or nearly straight flightpaths, the differential distance at ground level normal to the intended flightpath which results from a differential deflection of that flightpath is approximately equal to the arc length

$$dR = d\delta \int_t^{t_{Im}} V dt \tag{14}$$

where t_{Im} is the impact time. Since $dt = dh/(V\sin\gamma)$, Eq. (14) can be expressed

$$dR = d\delta \int_h^0 \left(\frac{1}{\sin\gamma}\right) dh = -d\delta\left(\frac{h}{\sin\bar{\gamma}}\right) \tag{15}$$

where

$$\frac{1}{\sin\bar{\gamma}} = \left(\frac{1}{h}\right)\int_0^h \left(\frac{1}{\sin\gamma}\right) dh \tag{16}$$

The first equalities of Eq. (12) combined with Eqs. (13) and (15) can be expressed

$$\begin{aligned} \dot{R}_x &= (g/2)(C_{L\alpha}\alpha A/W)(\rho V h/\sin\bar{\gamma})\cos\theta + \\ &\quad d\{[h/(V\sin\bar{\gamma})]\dot{x}\}/dt - \{1 + (g/2)[\rho h/(\beta\sin\bar{\gamma})]\}\dot{x} \\ \dot{R}_y &= -(g/2)(C_{L\alpha}\alpha A/W)(\rho V h/\sin\bar{\gamma})\sin\theta - \\ &\quad d\{[h/(V\sin\bar{\gamma})]\dot{y}\}/dt + \{1 + (g/2)[\rho h/(\beta\sin\bar{\gamma})]\}\dot{y} \end{aligned} \tag{17}$$

where $\beta = W/C_DA$ is the ballistic coefficient. The second equalities of Eq. (12) can be combined to obtain

$$\begin{aligned} r(\dot{\varphi})^2 &= g(L/W)\cos\epsilon \\ d(r\dot{\varphi})/dt &= g(L/W)\sin\epsilon \end{aligned} \tag{18}$$

where $r = (x^2 + y^2)^{1/2}$. Equation (18) can be combined with Eq. (8) to obtain

$$\epsilon = -\tan^{-1}\{(\dot{P} + \ddot{\epsilon})/[(P + \dot{\epsilon})(P + 2\dot{\epsilon})]\} \tag{19}$$

Equation (19) will be used to obtain a set of sufficient conditions which will aid in the integration of Eq. (17). The vehicle motion at constant P has already been described and yields

$$\epsilon = \dot{\epsilon} = \ddot{\epsilon} = 0 \tag{20}$$

Suppose that under the conditions of constant $\dot{P}$, $\epsilon \ll 1$, $\dot{\epsilon} \ll P$, and $\ddot{\epsilon} \ll \dot{P}$; then Eq. (19) yields $\epsilon \approx -\dot{P}/P^2$, $(\dot{\epsilon}/P) = 2\epsilon^2$, and $(\ddot{\epsilon}/\dot{P}) = -6\epsilon^2$. Therefore, Eq. (19) approximates Eq. (20) when

$$\dot{P} = \text{const}, \quad |\dot{P}|/P^2 \ll 1 \tag{21}$$

Equations (21) are sufficient conditions on P and $\dot{P}$ to insure vehicle motion close to that which would exist if P were constant. Assuming these expressions are satisfied at the end

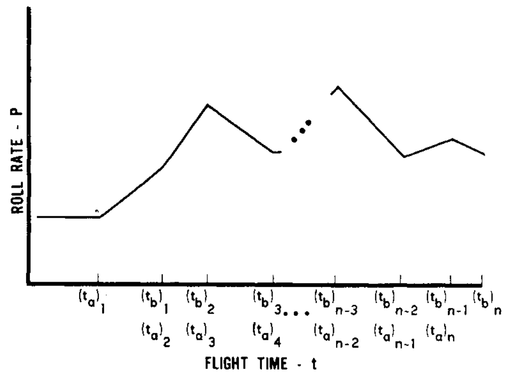

Fig. 3 Multiple linear roll rate variations.

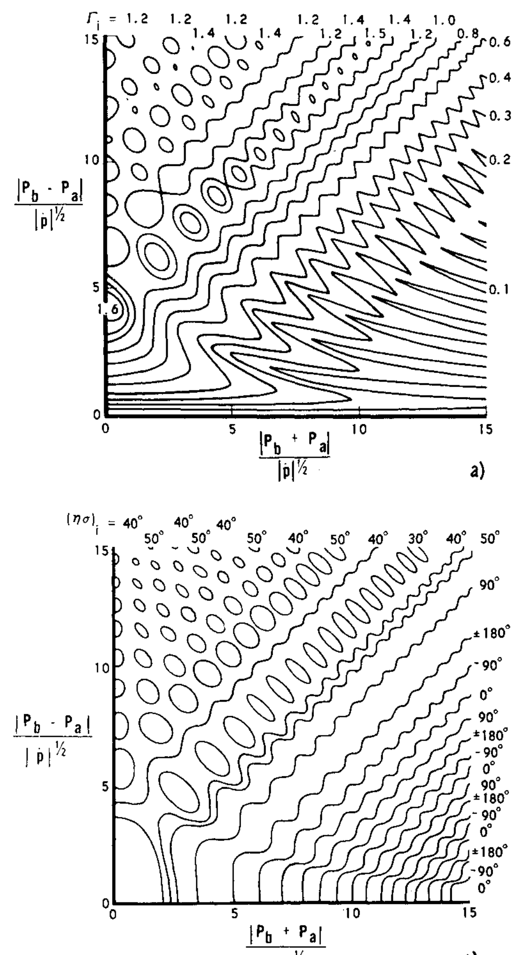

Fig. 4 Effect of the roll rate boundary conditions on impact.

points of the roll rate variation, and assuming

$$-(g/2)[\rho h/(\beta \sin\bar{\gamma})] \overset{\varphi}{<} 1 \tag{22}$$

$$(r_a + r_{Im})/(R_x^2 + R_y^2)^{1/2} \ll 1 \tag{23}$$

Equation (17) combined with Eqs. (3) and (8) can be integrated approximately over any interval of time that contains the interval over which P varies to obtain

$$R_x = \left(\frac{g}{2}\right)\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\right]_{\text{av}} \int_{t_a}^{t_b} \cos\theta\, dt - \left(\frac{g}{2}\right)\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\frac{\sin\theta}{P_b}\right]_b + \left(\frac{g}{2}\right)\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\frac{\sin\theta}{P}\right]_a \tag{24a}$$

$$R_y = -\left(\frac{g}{2}\right)\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\right]_{\text{av}} \int_{t_a}^{t_b} \sin\theta\, dt - \left(\frac{g}{2}\right)\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\frac{\cos\theta}{P}\right]_b + \left(\frac{g}{2}\right)\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\frac{\cos\theta}{P}\right]_a \tag{24b}$$

where $t_b - t_a$ is the time interval containing the roll rate variations assumed sufficiently small that the integration can be performed by replacing the variable vehicle and trajectory parameters with constant average values designated by the subscript *av*.

Assume that the prescribed roll rate history can be described with a series of linear variations, each extending over a small interval of altitude as illustrated in Fig. 3. The roll angle as a function of time for the first linear segment of this history can be determined by integrating Eq. (8)

$$\theta_1 = (\theta_a - P_a t_a + \tfrac{1}{2}\dot{P} t_a^2)_1 + (P_a - \dot{P} t_a)_1 + \tfrac{1}{2}\dot{P}_1 t^2 \tag{25}$$

The initial time of the variation t_a is arbitrary, provided the interval $t_b - t_a$ over which the variation occurs is correct. Therefore, choose

$$(t_a)_1 = (P_a/\dot{P})_1 \tag{26}$$

and Eq. (6) becomes

$$\theta_1 = [\theta_a - (\tfrac{1}{2})(P_a^2/\dot{P})]_1 + (\tfrac{1}{2})\dot{P}_1 t^2 \tag{27}$$

Differentiation of Eq. (27) yields

$$(t_b)_1 = (P_b/\dot{P})_1 \tag{28}$$

for the final time of the variation. In order to avoid a discontinuity when the second linear segment of the roll rate history begins, the roll angle at the end of the first linear segment must equal that at the beginning of the second. Therefore, from Eqs. (27) and (28),

$$(\theta_b)_1 = (\theta_a)_1 + (\tfrac{1}{2})(P_b^2 - P_a^2)/\dot{P}_1 = (\theta_a)_2$$

In general, the initial roll angle for any of the linear segments of Fig. 3 is

$$(\theta_a)_i = (\theta_a)_1 + \left(\frac{1}{2}\right)\sum_{j=1}^{i-1}\left[\frac{(P_b^2 - P_a^2)}{\dot{P}}\right]_j \tag{29}$$

and the roll angle is

$$\theta_i = (\theta_a)_i - (\tfrac{1}{2})(P_a^2/\dot{P})_i + (\tfrac{1}{2})\dot{P}_i t^2 \tag{30}$$

where

$$(t_a)_i = (P_a/\dot{P})_i \le t \le (P_b/\dot{P})_i = (t_b)_i$$

holds for each individual segment.

The effect of the roll rate history described by Eqs. (29) and (30) on impact error can be determined by substituting these expressions into Eqs. (24) and completing the integration. The result is

$$R_x = g\sum_{i=1}^{n}\left\{\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\right]_{\text{av}}\frac{\Gamma}{|\dot{P}|^{1/2}}\cos\left[\sigma + \theta_a - \left(\frac{\pi}{2}\right)\eta t_a'^2\right]\right\}_i + \frac{g}{(2\pi^{1/2})}\left\{\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\right]_a\frac{1}{|\dot{P}|^{1/2}}\frac{\sin\theta_a}{t_a'}\right\}_1 - \frac{g}{(2\pi^{1/2})}\left\{\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\right]_b\frac{1}{|\dot{P}|^{1/2}}\frac{\sin\theta_b}{t_b'}\right\}_n \tag{31a}$$

$$R_y = -g\sum_{i=1}^{n}\left\{\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\right]_{\text{av}}\frac{\Gamma}{|\dot{P}|^{1/2}}\sin\left[\sigma + \theta_a - \left(\frac{\pi}{2}\right)\eta t_a'^2\right]\right\}_i + \frac{g}{(2\pi^{1/2})}\left\{\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\right]_a\frac{1}{|\dot{P}|^{1/2}}\frac{\cos\theta_a}{t_a'}\right\}_1 - \frac{g}{(2\pi^{1/2})}\left\{\left[\frac{C_{L\alpha}\alpha A}{W}\frac{\rho V h}{\sin\bar{\gamma}}\right]_b\frac{1}{|\dot{P}|^{1/2}}\frac{\cos\theta_b}{t_b'}\right\}_n \tag{31b}$$

where

$$\Gamma_i = (\pi^{1/2}/2)\{[C(t_b') - C(t_a')]^2 + [S(t_b') - S(t_a')]^2\}_i^{1/2} \tag{32}$$

$$\sigma_i = \tan^{-1}\{[S(t_b') - S(t_a')]/(\eta[C(t_b') - C(t_a')])\}_i \tag{33}$$

$$C(t) = \int_0^t \cos\left[\left(\frac{\pi}{2}\right)x^2\right]dx, \qquad S(t) = \int_0^t \sin\left[\left(\frac{\pi}{2}\right)x^2\right]dx \tag{34}$$

$$(\theta_b)_i = (\theta_a)_i + \left(\frac{\pi}{2}\right)\sum_{j=1}^{i}[\eta(t_b'^2 - t_a'^2)]_j \tag{35}$$

$$(\theta_a)_i = (\theta_b)_{i-1} \tag{36}$$

$$(t_a')_i = \left(\frac{1}{\pi^{1/2}}\right)\left(\frac{P_a}{|\dot{P}|^{1/2}}\right)_i, \quad (t_b')_i = \left(\frac{1}{\pi^{1/2}}\right)\left(\frac{P_b}{|\dot{P}|^{1/2}}\right)_i \tag{37}$$

$$\eta_i = (\dot{P}/|\dot{P}|)_i \tag{38}$$

Fresnel's integrals, defined by Eqs. (34), are tabulated in Ref. 3. The functions of Eqs. (32) and (33) which contain these integrals, are presented in Figs. 4 in terms of the roll rate boundary conditions.

Since, by assumption, the trajectory parameters as functions of altitude for a vehicle experiencing lift and roll rate variation are the same as those for the zero-lift case, they are known or can be obtained independent of this analysis. If the zero-lift flight-path deflections caused by roll rate variation generate significant impact errors within the assumptions of this theory, they will occur at sufficiently high altitudes that $V \approx V_e$. Assuming this is true and for $|\gamma| \ge 20°$, an approximate trajectory solution which assumes that $\gamma = \gamma_e = \text{const}$ can be used to approximate the zero-lift vehicle velocity history. Assuming the exponential density relation,[4] $\rho = 0.0027\exp(-h/23500)$ the trajectory analysis of Ref. 5 yields

$$V = V_e\exp\{[1020/(\beta\sin\gamma_e)]\exp(-h/23500)\} \tag{39}$$

These expressions can be used to determine $g(\rho V h/\sin\gamma_e)$ of Eq. (31) approximately as

$$g(\rho V h/\sin\bar{\gamma}) \approx (V_e/\sin\gamma_e) f(h, \beta\sin\gamma_e) \tag{40}$$

where the trajectory function,

$$f(h, \beta\sin\gamma_e) = 0.0869h\exp\{(-h/23500) + [1020/(\beta\sin\gamma_e)]\exp(-h/23500)\} \tag{41}$$

is presented in Fig. 5.

At impact, the total distance that the mean flightpath has been deflected normal to the intended flightpath is the root-

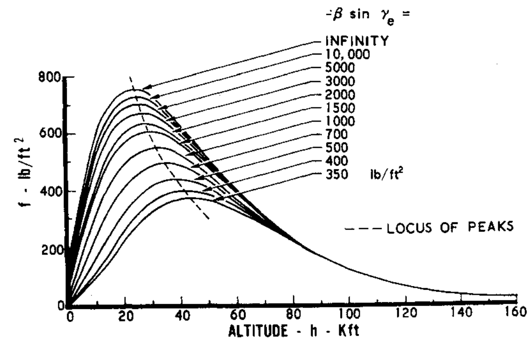

Fig. 5 Influence of the trajectory function of impact.

sum-square of Eqs. (31);

$$R = (R_x^2 + R_y^2)^{1/2} \quad (42)$$

If the cross range and down range directions are unspecified in the Newtonian coordinate system of Fig. 2, Eq. (31) describes the effect of a mean flightpath deflection which could have occurred in any direction about the zero-lift flightpath. The locus of all possible mean flightpaths that could occur as the result of a particular deflection magnitude forms a conical surface about the zero-lift flightpath. Assuming that the impact deviation is dominated by the deflection of the mean flightpath, the ellipse defined by the intersection of this conical surface with the ground plane relative to the impact point of the zero-lift flightpath defines the impact deviation possible as a result of the roll rate variation. The semiminor axis of the ellipse and maximum possible cross range deviation is

$$(CR)_{\max} = R \quad (43)$$

where R is defined by Eq. (42). The semimajor axis and maximum possible down range deviation is

$$(DR)_{\max} = -R/\sin\gamma_{Im} \quad (44)$$

Further, Eqs. (43) and (44) define the minimum and maximum impact deviation possible as the result of a particular mean flight-path deflection, respectively. If the cross range and down range directions are specified in the Newtonian coordinate system of Fig. 2, then

$$\theta_{id} = \tan^{-1}(R_y/R_x) \quad (45)$$

specifies the direction of the deflection with reference to Fig. 2, and, therefore, specifies where the impact point is located on the ellipse of all possible impact points.

A Single Linear Roll Rate Variation

In order to identify two important special cases, consider a single linear roll rate variation. For simplicity, choose the initial roll angle to be $\theta_a = (\pi/2)\eta t_a'^2$, and Eqs. (31) and (35) yield

$$R_x = [g/(2\pi^{1/2})][(C_{L\alpha}\alpha A/W)(\rho Vh/\sin\bar{\gamma}]_{av}(\eta/|\dot{P}|^{1/2}) \times \{(2\pi^{1/2}/\eta)\Gamma\cos\sigma + \sin[(\pi/2)t_a'^2]/t_a' - \sin[(\pi/2)t_b'^2]/t_b'\} \quad (46a)$$

$$R_y = -[g/(2\pi^{1/2})][(C_{L\alpha}\alpha A/W)(\rho Vh/\sin\bar{\gamma})]_{av}(1/|\dot{P}|^{1/2}) \times \{2\pi^{1/2}\Gamma\sin\sigma - \cos[(\pi/2)t_a'^2]/t_a' + \cos[(\pi/2)t_b'^2]/t_b'\} \quad (46b)$$

where

$$\Gamma\cos\sigma = (\pi^{1/2}/2)\eta[C(t_b') - C(t_a')],$$
$$\Gamma\sin\sigma = (\pi^{1/2}/2)[S(t_b') - S(t_a')] \quad (47)$$

$$t_a' = (1/\pi^{1/2})(P_a/|\dot{P}|^{1/2}), \quad t_b' = (1/\pi^{1/2})(P_b/|\dot{P}|^{1/2}) \quad (48)$$

and the Fresnel's integrals are defined by Eqs. (34).

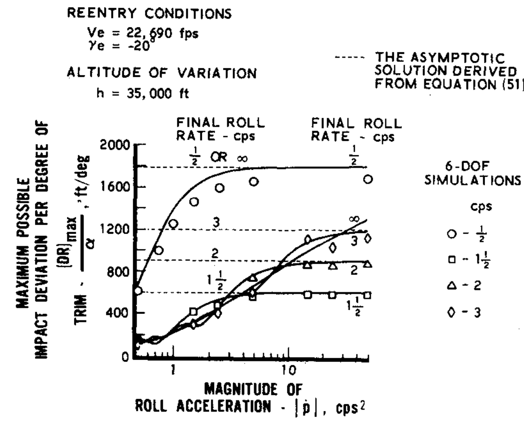

Fig. 6 Impact deviation caused by roll rate variation from 1 cps.

Roll through Zero

An important special case occurs when $P = 0$ at some instant during the roll rate variation. The effect of constant lift on the flightpath is greatest when $|P|$ is lowest, since the space-fixed impulse is greatest at this condition. If roll through zero occurs between initial and final roll rates of large magnitudes, but opposite signs, the influence of the roll rate boundary conditions on the impact error is small compared to the influence around $P = 0$. This is illustrated in Figs. 4, since for

$$|P_b + P_a|/|\dot{P}|^{1/2} \ll |P_b - P_a|/|\dot{P}|^{1/2} > 5$$

the functions Γ_i and σ_i become insensitive to changes in the coordinates. If these conditions are satisfied, t_a' and t_b' are sufficiently large that the Fresnel's integrals of Eq. (34) are approximately equal to their limiting values

$$S(t') \approx C(t') \approx (\tfrac{1}{2})(t'/|t'|)$$

and dominate Eqs. (46). Equation (42), using Eq. (46), can be expressed

$$-\sin\gamma_{P=0}[W/(C_{L\alpha}\alpha A)]|\dot{P}|^{1/2}R = (\pi/2)^{1/2}g(\rho Vh)_{P=0} \quad (49)$$

where the trajectory parameters are evaluated at the altitude of $P = 0$ since most of the lift impulse occurs at that condition. The approximate relation of Eq. (40) in Eq. (49) yields

$$-(\sin\gamma_e/V_e)[W/(C_{L\alpha}\alpha A)]|\dot{P}|^{1/2}R = (\pi/2)^{1/2}f(h_{P=0}, \beta\sin\gamma_e) \quad (50)$$

Roll Rate Variation at Infinite Rate

Another interesting special case is that for infinite roll acceleration or deceleration between finite roll rate boundary conditions. The limit of Eq. (46) as $|\dot{P}| \rightarrow \infty$ can be substituted into Eq. (42) to obtain

$$-\sin\bar{\gamma}_0[W/(C_{L\alpha}\alpha A)]|P_bP_a/(P_b - P_a)|R = (g/2)(\rho Vh)_0 \quad (51)$$

Equation (40) in Eq. (51) yields

$$-(\sin\gamma_e/V_e)[W/(C_{L\alpha}\alpha A)]|P_bP_a/(P_b - P_a)|R = (1/2)f(h_0, \beta\sin\gamma_e) \quad (52)$$

Equations (51) and (52) describe the effect of a discontinuity in the roll rate history from P_a to P_b at h_0 on the impact error. Such a variation violates the condition of this analysis defined by Eqs. (21). The example problem presented at the end of this analysis indicates, however, that this violation does not seriously compromise the results, at least for the particular case treated.

Results and Conclusions

The impact deviation resulting from roll rate variation and constant α has been computed for an example re-entry vehicle with the following physical characteristics: $\beta = 2000$ lb/ft^2, $W = 131$ lb, $I_r = 0.34$ slug ft^2, $A = 0.92$ ft^2, and $C_{L\alpha} = 0.034$/deg. The re-entry velocity and flight-path angle were 22,690 fps and $-20°$, respectively. Roll rate variations from 1 cps down to 0.5 cps and up to 1.5, 2, 3, and ∞ cps were considered. These variations were assumed to be the result of $|\dot{P}|$ between 0.4 cps^2 and 50 cps^2. The $\dot{P}$ was initiated at an altitude of 35,000 ft and remained constant until P had varied the prescribed amount. The magnitude of the maximum impact error possible per degree of trim was computed using Eq. (46) combined with Eqs. (42) and (44), and is presented in Fig. 6. Also, the asymptotic solution corresponding to infinite $|\dot{P}|$ was computed from Eq. (51) and included in Fig. 6.

The deviation of the impact point from the zero-lift impact point which occurs as a result of lift and roll rate variation has been obtained from various six-degree-of-freedom trajectory simulations and is included in Fig. 6. The results derived from the simple model developed herein compare typically to within 5% of the results obtained from the trajectory simulations, with none of the differences exceeding 13%.

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Missile Aerodynamic Predictions to 180°

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A method for predicting the static, longitudinal aerodynamic characteristics of typical missile configurations at zero roll angle (i.e., in a plus configuration) has been developed and programed for use on the IBM 7090 digital computer. It can be applied throughout the subsonic, transonic, and supersonic speed regimes to slender bodies of revolution or to nose-cylinder body combinations with low-aspect-ratio lifting surfaces. The aerodynamic characteristics can be computed for missile configurations operating at angles of attack up to 180°. The effects of control surface deflections for all modes of aerodynamic control are taken into account. The method is based on linear, nonlinear-cross-flow, and slender-body theories with empirical modifications to provide the high angle-of-attack capability. Comparisons of the theory with experimental data are presented to demonstrate the accuracy of the method.

Nomenclature§

AR = exposed aspect ratio
b = semispan of an aerodynamic surface including r, ft
C_D = total drag coefficient; components of C_D are indicated by subscripts b, base; c, cross flow; f, friction; i, induced; 0, total zero-lift; p, pressure; v, wave
C_f, C_{fc} = incompressible and compressible skin-friction coefficients
$C_{D_{fp}}$ = drag of a flat plate normal to the flow
C_L = total lift coefficient
$C_{L\alpha}$ = lift curve slope, rad^{-1}
C_m = total longitudinal pitching moment coefficient
C_r, C_t = root-chord and tip-chord of an aerodynamic surface, ft
d = body diameter at any station, ft; d_b, body base; d_N, nose-body juncture
f = spanwise location of the vortex which emanates from the forward surface, ft
h_A = height of the trailing vortex above the body center line at the aft surface center of pressure, ft
i = downwash interference constant
$(k_2 - k_1)$ = apparent mass factor
K, K' = linear lift interference factors due to α and δ
l = length, ft; l_B = body length; l_{REF} = arbitrary reference length, usually the maximum body diameter
l_T, l_W = lengths from nose tip to the intersections of the tail and wing leading edges, respectively, with the body, ft
M = freestream Mach number
m = cotangent of the leading edge sweep angle
r = radius of the body at any station, ft
Re = Reynolds number

Presented as Paper 70-981 at the AIAA Guidance, Control and Flight Mechanics Conference, Santa Barbara, Calif., August 17–19, 1970; submitted August 28, 1970; revision received January 11, 1971.

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§ The control surface is defined as the tail regardless of the mode of control; the fixed surface is defined as the wing (see Fig. 1).